USING TRANSMISSION RATIOS AND MODE SHAPES FOR OPTIMISING PASSIVE DAMPING

A broad range of mechanical engineering techniques, from smart design principles to advanced motion control, is available for achieving dynamic performance. Somewhere in the mix, however, the field of passive damping is often overlooked. This article attempts to extend the understanding of mechanical engineers towards thinking in terms of dynamics and mode shapes. To that end, analogies between stiffness and modal mass in terms of transmission ratios and their effect on system performance are presented. This may provide insight, not only for where to place passive dampers, but also more generally into how a control system 'feels' the different vibration modes.

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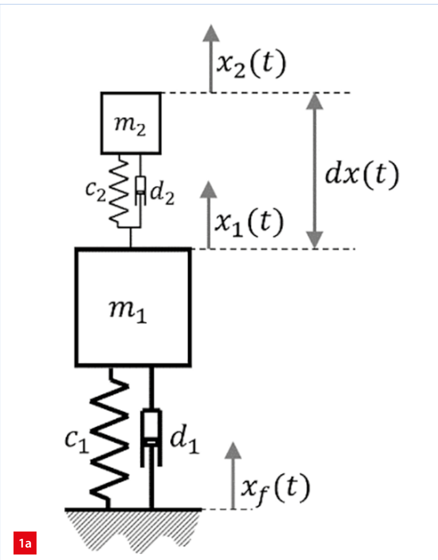
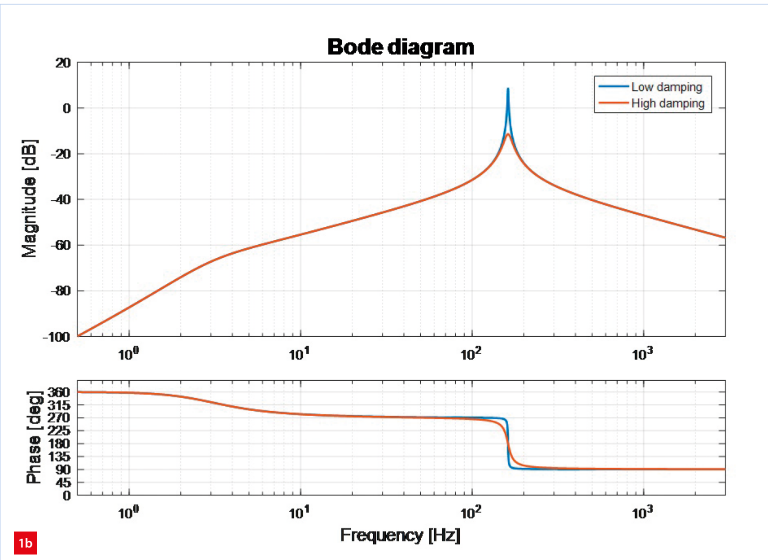
Introduction

The precision machine building community finds itself continuously facing new challenges in terms of requirements for dynamic performance. Precision machines have become faster and more accurate over time and this trend has not stopped, and will not stop in the future, as far as we can look. A large range of mechanical engineering techniques is available for achieving dynamic performance, ranging from sophisticated mechanical design principles – such as statically determined design – to complex and intelligent control and software solutions, such as advanced motion control and feedforward solutions. Somewhere in between these topics, and often stepped over, is the field of passive damping.

Passive damping has been studied since approximately the 1960s [1], but for many years was not usually applied in precision machine designs. This is in contrast to many other fields, such as structural engineering and aerospace engineering, where passive dampers have been integrated into designs for many decades, to counteract disturbances from wind, traffic, earthquakes, etc. and effectively limit the resulting deformation – and thereby, stress – in these





The effect of damping on machine dynamics.
(a) Dynamic model with dx(t) as point of interest (POI), which is the relative displacement output between mass 1 and mass 2.
(b) The Bode diagram shows the transmissibility from floor vibrations to this POI: $dx(t)/x_f(t)$.
The difference in damping value is visible at the resonance frequency around 160 Hz.

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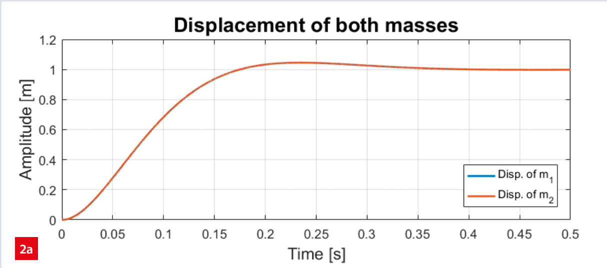
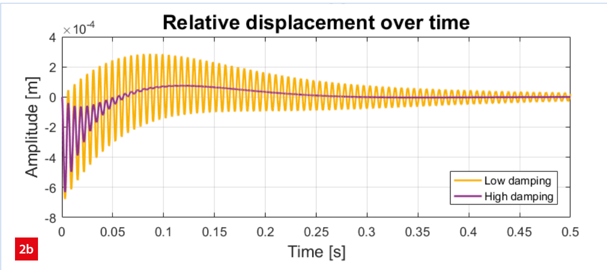




Responses of the system of Figure 1.
(a) Displacement of masses 1 and 2.
(b) Relative displacement between the two masses, at low and high damping values for the resonance at 160 Hz.

structures. The cause of this relatively late application of damping in the field of precision engineering seems to have a relation to the required accuracy of the high-tech systems involved. Accuracy, but primarily repeatability and reproducibility, need to be high, in contrast with the structures in other engineering fields that used damping much earlier. For these structures, mainly deflection was important, which relates directly to stress and safety factors. In the field of precision engineering, however, this is significantly different, as the focus lies on design for stiffness, low hysteresis, low friction, etc.
Over the last two decades, slowly but steadily, damping has been adopted in the field of precision engineering, enabled by improved computational power and ease of dynamic modelling. Material models have become more accurate [2], and experience has been gained in how to apply these materials in precision designs, as well as in dealing with the results in terms of system characteristics. Currently, this has resulted in multiple passive damper solutions in the field of precision machine design, even to being implemented in sub-nanometer precision machines.

The effect of damping

To zoom in on the topic of damping, we will divide precision machines into two categories, the first being fast machines that move quickly and need short settling times. These machines are typically limited in their performance by the high-frequency dynamics, which restricts the bandwidth of the feedback control loop and introduces transient oscillations in the settling phase (after the accelerations have ended). The second category includes machines that require good standstill performance.

In this case, the flexible dynamics is also problematic, as it is typically excited by various vibration sources, such as floor vibrations, acoustics, noise from electronics, etc.

In both cases, passive damping can help improve performance by adding damping to the flexible dynamics. For the first category of machines, this leads to short settle times, because the kinetic energy is dissipated more quickly. For the second category (standstill performance), higher damping values lead to less amplification of the oscillations at resonance frequencies, resulting in smaller steady-state vibration amplitudes.

As an example, Figure 1 presents a dynamic model of a machine. The input is – for simplicity – a random floor displacement spectrum (white noise), and the output is the relative displacement at the point of interest (POI), which is the position difference between the two masses ($dx(t)$ = relative output). The transmissibility from floor displacement to relative displacement at the POI is given in Figure 1b, which clearly shows the effects of the vibration isolation characteristics at 3 Hz and the internal dynamics at 160 Hz. The blue curve shows the transmissibility for low modal damping on the flexible dynamics, the red curve for a tenfold increase of the modal damping. The result is a lower amplification factor (a lower resonance peak).

Figure 2 shows a step response of the two masses of Figure 1a in the upper plot and the relative displacement in the lower plot. When the modal damping of the resonance at 160 Hz is changed, the upper plot hardly changes, because its characteristics originate mainly from the isolation system at 3 Hz. However, the lower plot shows a significant difference in settling time. Figure 2b shows the effect of this damping increase, at the resonance at 160 Hz, on the POI and in the time domain. The increased decay rate of the oscillation is clearly visible.

Note that this difference in vibration amplitude is caused by the increased damping of the internal dynamics only. In addition, for motion-control systems it is the increase of damping at resonance frequencies that enables higher bandwidths (open-loop cross-over frequency at 0 dB). As damping is increased, the resonance peaks are attenuated (see Figure 1b) and higher feedback gains can be applied with equal stability margins.

Increasing natural frequencies first

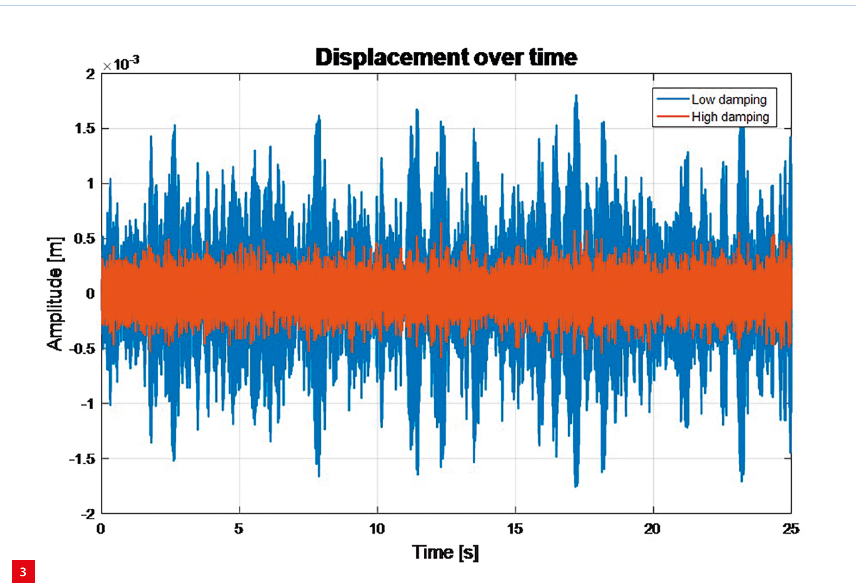
The field of passive damping adds a tool to the mechanical designer's toolbox. Once a mechanical design has been created according to the rules of precision engineering to maximise stiffness and minimise moving mass (i.e. maximise natural frequencies), damping can help to further improve performance.

Increasing the natural frequency basically solves the same problem as increasing the damping, is usually much simpler to implement and does not require additional calculational tooling and components such as dampers. Therefore, this order of engineering actions makes perfect sense. In Equation 1, the amplitude of a damped oscillation is shown as a function of time, where A_i is the initial vibration amplitude of the oscillation, ζ_i is the modal damping, ω_i is the natural frequency and φ_i is the phase shift of the oscillation.

$$x_i(t) = A_i e^{-\zeta_i \omega_i t} \cdot \sin(\omega_i t + \varphi_i) \qquad (1)$$

The first part ($A_i e^{-\zeta_i \omega_i t}$) describes the envelope of the sinusoidal oscillation over time. The expression in the exponent ($\zeta_i \omega_i t$) describes the rate of the amplitude decay as a function of time. This so-called exponential decay rate is influenced by the damping ratio ζ_i to dissipate energy in an oscillating system and equally by the natural frequency ω_i. By two times faster oscillations, settling time is shortened in the same way as by a twofold increase of the relative damping. This explains the need for high natural frequencies in a mechanical design. In addition, this high natural frequency helps to reduce the initial setpoint-induced vibration amplitude [3].

The second point is a more practical point and concerns the fact that damping is usually created by designing a damping device that makes use of a linear viscoelastic (LVE) material; see the text box. A specific group within the LVE materials are the rubbers, which are usually applied for damping applications around room temperature. In general (exceptions are possible), rubbers tend to show increasing damping values at higher frequencies, typically in the frequency range (100 Hz to a few kHz) that mechanical engineers are dealing with. This implies that the application of a rubber becomes more effective once a sound mechanical design has been created with high natural frequencies. To summarise: although there is new paradigm of designing for damping to improve performance, first the old paradigm of designing for high natural frequencies has to be pursued.



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Time domain simulations that clearly show the difference in displacement between the system with low damping (blue) and high damping (red).

Linear viscoelastic materials

Linear viscoelastic materials (LVE) show linear frequency-dependent damping. The damping is typically low for low frequencies and increases by orders of magnitude with increasing frequency. Beyond a certain frequency – depending on the material – the damping drops and ultimately vanishes. Control engineers typically deal with these characteristics every day when designing a lead filter, which shows the same characteristics as an LVE material.

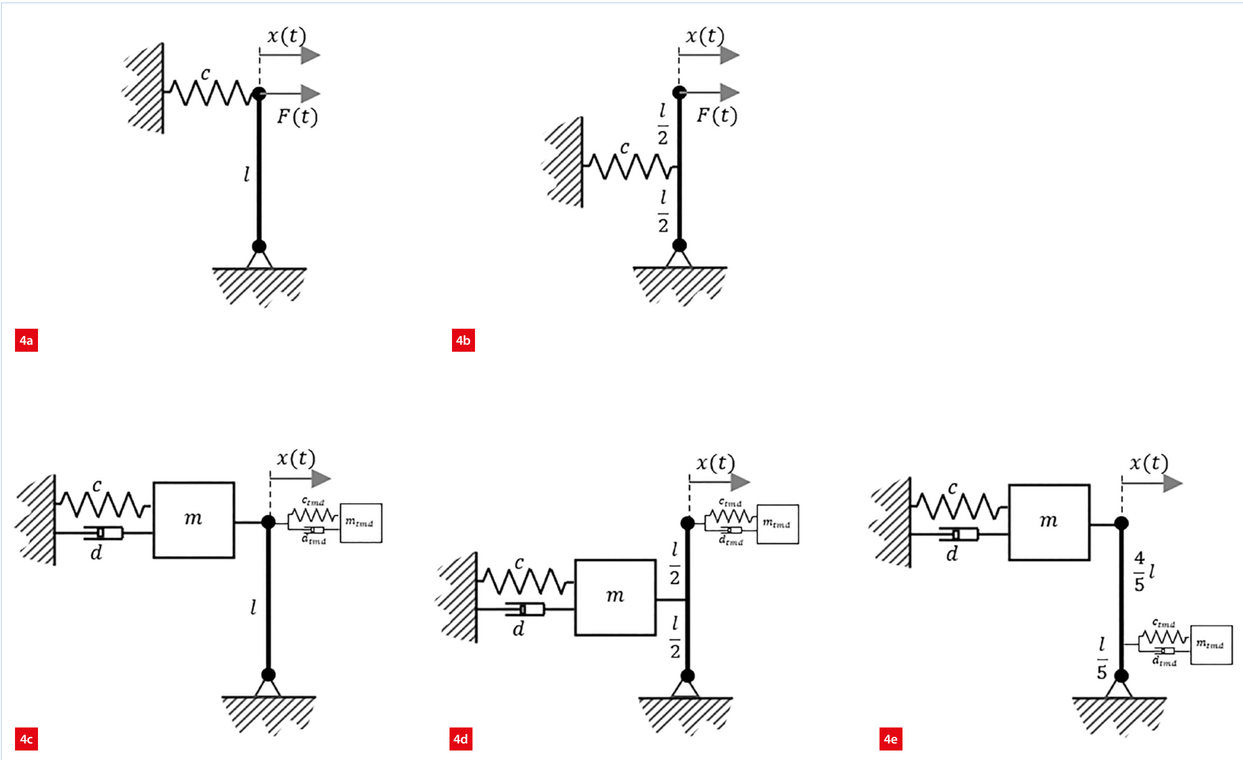
Although this feels as if it is 'nonlinear' material behaviour, it is not. This characteristic behaviour can be approximated with linear equations – mimicking a combination of springs and dampers (Maxwell model) – and, therefore, is linear system behaviour. Control engineers are familiar with this: a simple lead filter is a component from linear control theory.

Optimisation of damping

Increasing the damping of a mechanical structure in the field of precision engineering typically implies the design of additional, artificial damping mechanisms that increase the damping at the resonance frequencies; see the text box on the next page. These devices, such as tuned mass dampers (TMDs) and constrained layer dampers (CLDs), are well known and have been extensively described in literature [4,5], with many different variations on these topics. Analytical solutions exist for relatively simple problems, such as finding the optimal damping for a specific resonance frequency. For more complex problems, such as optimising the damping over a range of resonances, or optimising for other criteria (i.e. control bandwidth) including the behaviour of dampers, optimisation algorithms can help solving these questions [2].

Damper placement

An important question for engineers who want to apply damping to a structure, which has not been discussed extensively in literature yet, is where dampers should be placed to maximise performance. This depends on the type of damper, and for the remainder of this article we will dive into the placement of TMDs.



Analogy between design for high stiffness and optimal TMD placement.
(a) Effective stiffness with a transmission ratio of 1.
(b) Transmission ratio of 2, defined as output displacement x(t) over spring displacement, leading to a four times lower stiffness at the output.
(c) TMD acting on a mass-spring-damper via a transmission ratio of 1.
(d) TMD acting on a system with a transmission ratio of 2, leading to an effectiveness increase for the TMD.
(e) Transmission ratio of 1/5, leading to a very ineffective lever ratio, hence the TMD is 25 times less effective as compared to the case with a ratio of 1.

Damping devices for improving performance

Different methods of increasing damping exist, with the simplest option being to increase material damping. However, this is often hard to realise because structural materials are usually selected for other mechanical properties, and high stiffness usually implies low damping, unless composite materials are applied [6].
The alternative is to add a damping device:

- A tuned mass damper (TMD) device is specifically tuned to damp a certain natural frequency. It is very effective, but sensitive to parameter variations.
- A constrained layer damper (CLD) uses the deformation (strain) of a certain surface to add damping. It consists of a rubber (constrained) layer and a metal (constraining) layer, which is designed such that shear deformation is passed on to the rubber layer, which increases the damping.
- A robust mass damper (RMD) looks like a TMD from the outside, but uses a much higher damping value. The result is that it increases the damping at many resonances. Its disadvantage is the complex mathematics or models that need to be studied; simple analytical equations do not exist for these devices.

To be effective, dampers in general need velocity difference across them. TMDs need input displacement and velocity, so these add-on devices should be placed at locations where the displacement is maximal for the mode shape that needs to be damped, which implies that a certain understanding must be gained of the mode shapes present in the system at hand and how they manifest themselves in a dynamic system.

A TMD can be seen as an intrinsic local ‘control loop’, picking up the displacement and velocity at its mounting position and, in response to this, applying a reaction force back to the main structure. This description of a TMD is the mechanical equivalent of a local motion-control loop. Studying the optimal location for a TMD creates understanding about capabilities for dealing with resonances similar as in a feedback control loop. This is referred to as the observability and controllability properties of a structure [7].

In addition, the principles of damper placement are quite comparable to the rules we apply to design for high stiffness. Figure 4 shows the effect of a transmission ratio on the stiffness c that is felt at the output at point A, where force $F(t)$ is applied. In the case of Figure 4a, this is simply the stiffness c [8]. In Figure 4b, a transmission ratio of 2 is applied, meaning that the displacement $x(t)$ at point A equals two times the elongation of the spring attached at point B. This results in a total transmission ratio of 4 (defined as output divided by input), since both the force on the spring and the displacement at the output scale with the transmission ratio, which leads to doubling the elongation of the spring and a fourfold increase of the displacement at the output.

Transmission ratio impact

So, in general, the effective stiffness scales with the transmission ratio squared. This principle is well known to mechanical engineers, and an equivalent principle applies for damper placement on a mechanical system. This is shown in Figures 4c to 4e.

Figure 4c shows a lumped-mass-spring-damper system with one translational mode shape. The undamped natural frequency is determined by the spring and the mass, and a TMD is inserted to add damping to this mode. As a rule of thumb and supported by practical cases, 10% of the main mass m can be used as an educated guess for m_{tmd} to design an effective damper based on tuning rules according to [4].

Figure 4d shows the same same TMD mass attached to the main mass, but now via a transmission ratio equal to 2. The displacement $x(t)$, fed into the damper, equals the displacement of mass m multiplied with the transmission ratio. The force produced by the damper is also multiplied by the transmission ratio before it acts on mass m. Effectively, the TMD mass now feels only a fraction of the main mass m. This implies that the transmission ratio squared is also present in this case, leading to the conclusion that the TMD mass of Figure 4d can be 25% of that in Figure 4c with exactly the same effectiveness on a system level.

Likewise, the effectiveness of the TMD in Figure 4e decreases by a factor of 25 with respect to Figure 4, hence a 25 times mass increase is required for the TMD to obtain the same effectiveness. Between Figure 4d and Figure 4e the efficiency difference is even a factor of 100. When the TMD is moved even further towards the pivot point, its effectiveness on the main system decreases to zero. Here, the modal mass that is felt by the TMD goes to infinite.

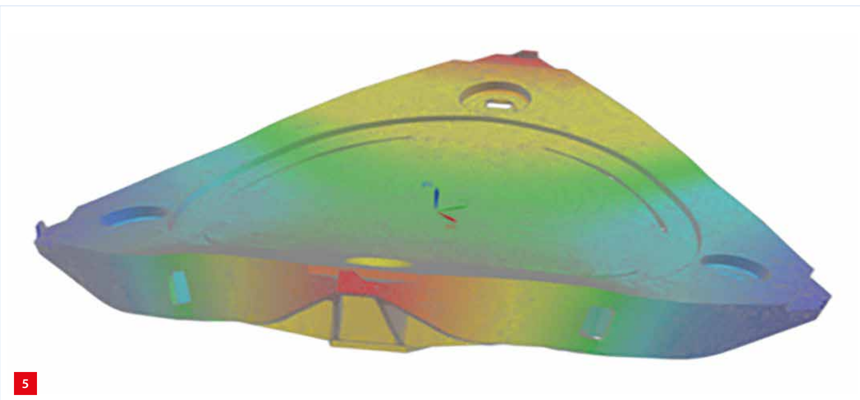
An 'inverse way' of thinking about the system is to look at it from the TMD side: the effectiveness of a TMD changes as a function of its location (the transmission ratio). This implies that the main mass that is felt by the TMD (called modal mass) changes with the position of the TMD. This is exactly what happened in Figure 4 when the transmission ratio changed; it affects the apparent modal mass of the main system as felt by the TMD. This observation is key to understanding mode shapes and the way they interact with other components in the system.

Mode-shape representation

Usually, the mode shapes of precision designs are more complex than the versions in Figure 4 and show complex dynamics of multiple components in different directions. Nevertheless, one particular mode can be modelled as a single mass-spring-damper system in order to study the effectiveness of a TMD implementation. In this case, the transmission ratio is not visible as shown in Figure 4, but is present in the form of the mode shape. An example of a more complex mode is shown in Figure 5, which is the first flexible mode (torsion mode) of a cordierite 450-mm wafer stage.

A simplification of this is presented in Figure 6, which shows the torsion mode for a flat plate. The natural frequency is different, but the same principles for damper placement apply. The goal is to increase the modal damping of this mode. The effect of different TMD locations is shown in Figure 6. Figures 6a, 6b and 6c show the effect of damper placing at the preferred location for this mode shape, where modal displacement is maximal. At this plate corner, the modal factor (or transmission ratio) is 2.11. This leads to a relatively high transmission ratio (Figure 6b), which is fundamentally equivalent to applying the TMD to a small main mass (modal mass). The modal mass m in this case is 0.225 kg.

In Figure 6d, the TMD has been moved towards the centre of the plate. The displacement is approximately 2.8 times less, giving a modal factor of 0.75 and leading to a much less effective transmission ratio, as shown in Figure 6e. This is essentially the same as applying the TMD to an eight times higher modal mass, visualised schematically in Figure 6f. This example shows that damper placement is key to effective suppression of vibration amplitude at a particular resonance frequency with corresponding mode shape, thereby increasing the modal damping of a structure.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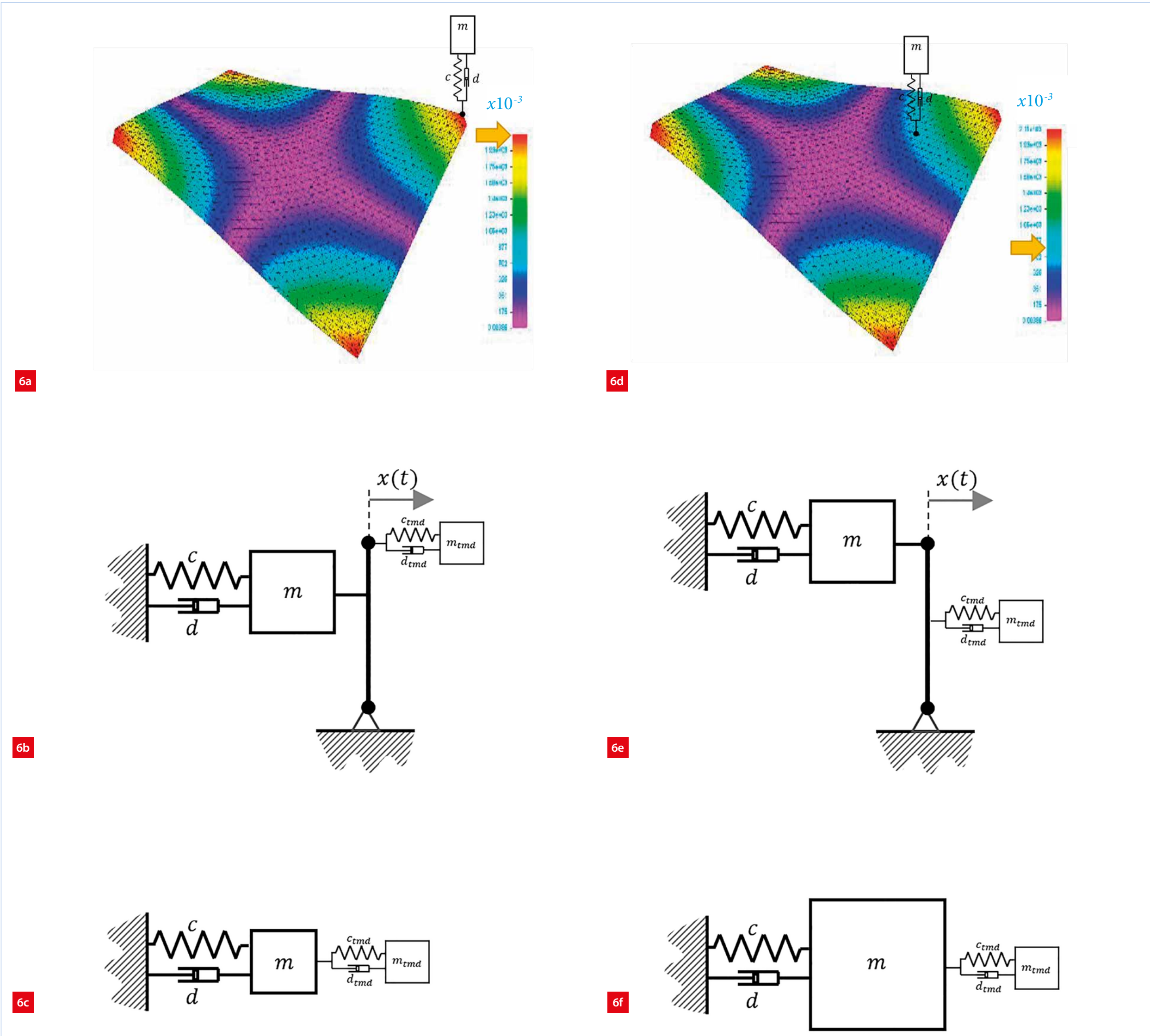
First flexible mode of a 450-mm wafer chuck (mass about 23 kg). The natural frequency, calculated using finite-element analysis, is approximately 1530 Hz. This mode, called the torsion mode, mainly shows displacement in vertical direction.



TMD placement on a structure. The upper figures (a and d) show the application of a TMD on a plate in two different positions. The middle figures (b and e) show the transmission ratio induced by the mode shape. The lower figures (c and f) show the representative dynamic model. Note the difference in effective mass in the lower two figures.

In pursuing optimal performance for a particular TMD mass, it is key to understand and to apply this approach correctly. A vibration mode can be damped at any location on the structure as long as there is modal displacement, but the effectiveness varies significantly. The induced change in damping will manifest itself in the total mode shape, regardless of the position where a measure of this mode is taken. Practically, this means that if a certain mode is damped at one corner, the other corner will experience the same amount of modal damping, because a mode (mode shape) is damped, not a node (a certain location on the structure). Consequently, a certain mode shape leading to high resonances in a control loop from actuator to sensor, can often can be damped more easily by a TMD at another location (not necessarily the location of the sensor), where the displacement of this mode is maximal. At this location the TMD-mass can be relatively small.

Conclusion

To summarise, the larger the modal factor (visualised as displacement in FEM results) the larger the transmission ratio (modal factor) and the lower the TMD mass required to be sufficiently effective. This implies that a TMD needs to be located at a position where the displacement – and thereby the velocity – is maximum for a certain mode shape. Intuitively, this feels right for engineers. The opposite way of thinking is usually a bit more counterintuitive: the larger the displacement at the TMD mounting location, the smaller the modal mass of the main system is. It is apparent, based on the example above, that the knowledge to understand modes in a broad sense is the key to an effective design for damping. Understanding this makes life relatively simple, because all mode shapes can be transformed into single-DoF (degree of freedom) dynamic systems, from which damper effectiveness can be calculated relatively simply.

This article has made an attempt to connect to the understanding of mechanical engineers and extend it towards thinking in terms of dynamics and mode shapes and attenuating vibration amplitudes through passive damping. To that end, the analogies in terms of transmission ratios and their effect on system performance have been used: the same quadratic equation for transmission ratios holds for stiffness and mass. In general, understanding this matter gives clear insight into not only where to place the dampers, but also in a broader sense into how a control system 'feels' the different vibration modes.

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