Elastic elements generating a

In mechatronic machines with a high positioning accuracy (i.e. submicron positioning), it is often necessary to have a stiff connection between the machine components. It can be deduced from the mechatronic model of a machine of this type that a certain connection between components has to be stiff (but not necessarily strong). A defined pretension force between the components is needed to optimise connections for (contact) stiffness. The design principle of a harmonica bush offers an elegant solution.

• *Paul Brom* •

It is not always possible to realise a connection by simply bolting one component to another. Optical components, for example, can be fragile or susceptible to induced (elastic or even plastic) deformations. These unwanted deformations are typically the result of the moments and forces that are generated when fixing the bolts using tools such as wrenches or screwdrivers. The mounting torque that is needed to generate the pretension force has to be eliminated from the design. This can be done by using an extra element between the bolt head and the component that will generate the pretension force without using torque. By using elastic elements, the pretension force can be generated and applied to the components in a controllable and defined fashion.

Harmonica bush

A good alternative for an elastic element would be a stack of disc springs. Disc springs are cheap and compact but have the disadvantage that the force level depends on tolerances of the discs in the stack and is hence less controllable. In one particular case, the requirement for a controllable pretension was included in the design at a later stage, which meant that there was only a limited volume left in which to create a solution. There is a good example of an elastic element that is used for generating pretension force, namely a harmonica bush ([1], page 130); see Figure 1. This principle was used in the design. Graphs from [1] can be used to determine the optimal dimensions of a harmonica bush; see Figure 2.

Definitions

I = moment of inertia [m^4]
I_p = polar moment of inertia [m^4]
C_z = axial stiffness [N/m]
M_w = torsion moment [Nm]
M_b = bending moment [Nm]
E = modulus of elasticity [N/m^2] (Young's Modulus)
G = shear modulus [N/m^2]
F = force [N]
r = radius [m]

for

pretension force

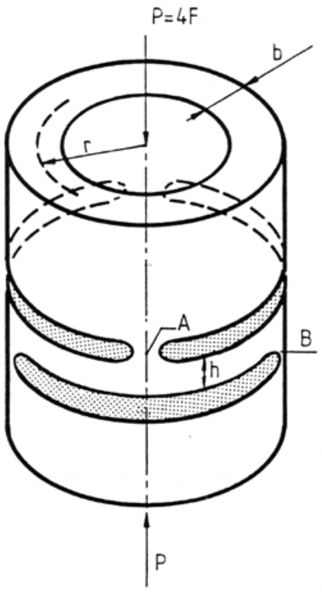

Figure 1. The harmonica bush design principle [1].

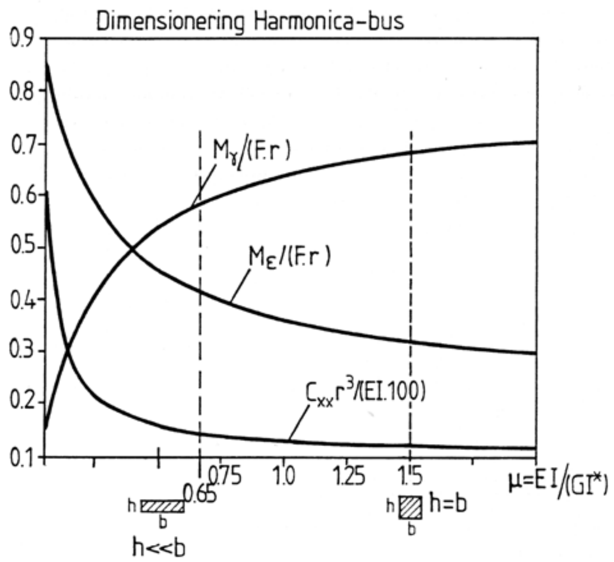

Figure 2. Dimensioning graphs for a harmonica bush, [1].

Formulas

It proved impractical to check these graphs for every iteration in the parameter set (height, width, radius, number of windings). The exact formulas for these graphs were not available, so approximations were generated for use in optimizing the design. By using fit-software on data points taken from the graphs, it was possible to convert the graphs into formulas.

The graph depicting the axial stiffness C_z is of shape:

$y_{\prime}^2 = a + (b/\sqrt{x_{\prime}})$;
with a = 0.0032976225 and b = 0.013599364.

Also valid is: $y_{\prime} = (C_z * r^3)/(E*I*100)$; $x_{\prime} = (E * I)/(G * I_p)$.

Using these formulas, an expression for C_z can be found and pasted in a spreadsheet.

The graph depicting the bending moment M_b is of shape:

$y_{\prime\prime} = c + d * e^x + f/x_{\prime\prime}$;
with c = 0.66689208; d = 0.0099309907 and
f = 0.078606373.

Also valid is: $y_{\prime\prime} = M_b /(F* r)$; $x_{\prime\prime} = (E * I)/(G * I_p)$

Thus an expression for M_b can be found.

The graph depicting the torsion moment M_w is of shape:

$y_{\prime\prime\prime} = g + j *(\ln x_{\prime\prime\prime})^2 + k * \ln x_{\prime\prime\prime}$;
with g = 0.3512396; j = 0.027846045 and k = 0.11052021.

Also valid is: $y_{\prime\prime\prime} = M_w /(F* r))$; $x_{\prime\prime\prime} = (E * I)/(G * I_p)$

Using these formulas, the expression for M_w can be found.

Each of the four elastic elements of one winding is actually a curved leafspring.
The I (bending) and the I_p (torsion) of a leafspring can be determined using Roark's Formulas for Stress & Strain ([2], Table 10.7, page 401).

If w *is* width and h is height of the leafspring and also $w = 2*m$ and $h = 2*n$ are calculated, then the expressions for I and I_p are:

$$I = 1/12 * (w * h^3);$$

$$I_p = (m*n^3) * (16/3 - 3{,}36 * \frac{n}{m} * (1 - \frac{n^4}{12 * m^4})).$$

Using these formulas, expressions for I and I_p can be found and pasted in the spreadsheet.

Pretension

The pretension level of the completely squeezed bush can be calculated. The dimensions of the slots in the harmonica bush are such that if the bush is squeezed completely, the desired pretension level is reached. The accuracy of the slots (made by wire eroding) is a direct measure for the accuracy of the pretension level. The maximum stress in the bush can be calculated using the bending and torsion moments. Next, all the parameters and formulas were pasted in a spreadsheet. Using the spreadsheet makes it possible to rapidly iterate the design of the harmonica bush to an optimum for the given volume and requirements. The design was verified with finite-element analysis (FEA) at a later stage. (Note: the iterating process for achieving the optimum design with the spreadsheet is quicker than with FEA).

Final design

A final design is shown in Figure 3. As an added feature, there are four holes that are partially threaded at the bottom. Using four small screws, the harmonica bush can be squeezed and locked prior to mounting in the machine; see Figure 4. When mounting the bush in the machine, the central bolt must be mounted hand tight (low distortion) on the counterpart and the bush. Then the four small screws are loosened and the pretension that was locked in the bush is subsequently transferred to the counterparts. It is possible that loosening the four small screws can also introduce unwanted moments and forces into the construction. But these will be much smaller as the central bolt will only be stretched in a defined manner. Even these final small disturbances can be removed if an anti-torque leafspring is used underneath the four screws.

Author's note

Paul Brom is a mechanical designer.

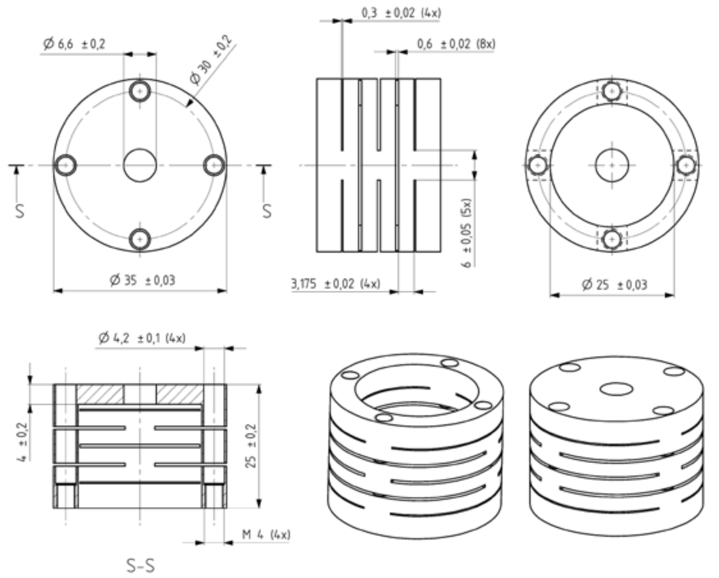

Figure 3. A final design of the harmonica bush.

Figure 4. The harmonica bush, on the right in its 'locked state' with the four screws that are used for controlling the pretension.

References

[1] Koster, M.P., Constructieprincipes (Mechanical Design Principles, in Dutch), third edition, Twente University Press, 2000.

[2] Young, W.C., Budynas, R.G., Roark's Formulas for Stress & Strain, seventh edition, McGraw-Hill, 2002.